# DIFFRACTION OF A NONSTATIONARY PRESSURE WAVE ON A MOVING PLATE

(DIFRAKTSIA NESTATSIONARNOI WOLNY DAVLENIIA NA PODVIZHNOI PLASTINE)

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The problem of the plane diffraction of a nonstationary acoustic pressure wave on a fixed infinite plate of a given width has been solved by the method of successive approximations in [1]. In the general case, under the action of an impinging pressure wave, the plate will start to move which considerably complicates the problem. Below, a method developed by Fok [2] is used to obtain an exact solution by quadratures of that problem, with account of the displacement of the plate; the pressure within the liquid is determined and also the force exerted by the liquid on the plate is calculated; the equation for the plate motion is set up and its solution for an arbitrary instant of time is given; an explicit relation between the shape of the impinging wave and the mode of plate motion is established; the initial transient of the plate motion within a time interval equal to twice the diffraction time is analyzed.

**1.** Assume that a plane wave having a pressure profile given by

$$p = P\left(t - \frac{z}{c}\right), \qquad P(\xi) \equiv 0 \text{ for } \xi \leqslant 0$$

meets at a time $t = 0$ a thin rigid plate $-l/2 \leqslant x \leqslant l/2$, which is situated in the plane $z = 0$ (Fig. 1).

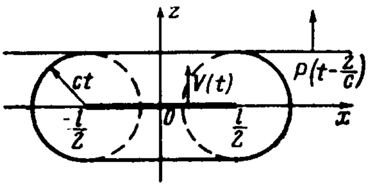


Fig. 1.

Under the impact of the wave the plate will start moving with a velocity $v_z = V(t)$ where $V(0) = 0$. We shall determine the pressure within the liquid for $t > 0$. To that end one needs to solve the equation

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} \tag{1.1}$$

for conditions

$$\frac{\partial p}{\partial z} = -\rho_0 \frac{\partial v}{\partial t} \quad \text{for } z = 0 \tag{1.2}$$

$$p = P\left(t - \frac{z}{c}\right) \quad \text{for } t \leqslant 0 \tag{1.3}$$

Here $v(x, t)$ is the velocity in direction of $z$-axis within the plane $z = 0$, $c$ and $\rho_0$ are the speed of sound and the density of stagnant liquid, respectively.

Let us denote by $p_-(x, z, t)$ and $p_+(x, z, t)$ the pressure for $z < 0$ and $z > 0$ respectively. The solution will be taken in the form [3,4]

$$p_-(x, z, t) = P\left(t - \frac{z}{c}\right) + P\left(t + \frac{z}{c}\right) - \frac{c\rho_0}{\pi}\frac{\partial}{\partial t}\int_0^{\vartheta_+} d\tau \int_{\chi_-}^{\chi_+} \frac{v(\xi, \tau)\, d\xi}{\sqrt{c^2(t-\tau)^2 - (x-\xi)^2 - z^2}}$$

$$p_+(x, z, t) = \frac{c\rho_0}{\pi}\frac{\partial}{\partial t}\int_0^{\vartheta_-} d\tau \int_{\chi_-}^{\chi_+} \frac{v(\xi, \tau)\, d\xi}{\sqrt{c^2(t-\tau)^2 - (x-\xi)^2 - z^2}} \tag{1.4}$$

$$\vartheta_\pm = t \pm z/c, \qquad \chi_\pm = x \pm \sqrt{c^2(t-\tau)^2 - z^2}$$

It can be readily verified that functions $p_-$ and $p_+$ satisfy Equation (1.1) and conditions (1.2) and (1.3).

We shall adopt dimensionless quantities

$$x_1 = \frac{2x}{l}, \qquad t_1 = \frac{2ct}{l}, \qquad v_1 = \frac{v}{c}, \qquad p_1 = \frac{p}{\rho_0 c^2}, \qquad P_1 = \frac{P}{\rho_0 c^2}$$

and we shall omit in the following the index 1.

The pressure drop along the plate is

$$\Delta p = p_-(x, 0, t) - p_+(x, 0, t) = 2\left[P(t) - \frac{1}{\pi}\frac{\partial}{\partial t}\int_0^t d\tau \int_{x-(t-\tau)}^{x+(t-\tau)} \frac{v(\xi, \tau)\, d\xi}{\sqrt{(t-\tau)^2 - (x-\xi)^2}}\right] \tag{1.5}$$

Outside the plate for $z = 0$ the pressure is a continuous function

$$p_-(x, 0, t) = p_+(x, 0, t) \quad \text{for } |x| > 1 \tag{1.6}$$

The function $v(x, t)$ entering solution (1.4) is unknown off the plate. We determine that function for $|x| > 1$ using relation (1.6). To that end we solve the integral equation of the first type

$$\frac{\partial}{\partial t}\int_0^t d\tau \int_{x-(t-\tau)}^{x+(t-\tau)} \frac{v(\xi, \tau)\, d\xi}{\sqrt{(t-\tau)^2 - (x-\xi)^2}} - \pi P(t) = 0 \quad \text{for } |x| > 1 \tag{1.7}$$

A similar equation was solved in [5].

We shall apply to (1.7) the Laplace transform with respect to $t$ and denote by $P^*(\lambda)$, $V^*(\lambda)$ and $\phi(x, \lambda)$ the transforms of the functions $P(t)$, $V(t)$ and $v(x, t)$ respectively. Considering the $\phi(-x, \lambda) = \phi(x, \lambda)$ we obtain

$$\lambda \left\{ \int_1^\infty [K_0(\lambda|x-\xi|) + K_0(\lambda|x+\xi|)] \varphi(\xi,\lambda)\, d\xi + V^*(\lambda) \int_{-1}^{1} K_0(\lambda|x-\xi|)\, d\xi \right\} - \pi P^*(\lambda) = 0 \qquad (x > 1) \tag{1.8}$$

Here $K_0$ is the MacDonald function. For sake of convenience we shall put

$$x - 1 = x_1, \qquad \varphi(x_1 + 1, \lambda) = \varphi_1(x_1, \lambda)$$

and we shall omit the index 1; then Equation (1.8) assumes the form

$$\lambda \left\{ \int_0^\infty [K_0(\lambda|x-\xi|) + K_0(\lambda|x+2+\xi|)] \varphi(\xi, \lambda)\, d\xi + V^*(\lambda) \int_0^2 K_0(\lambda|x+\xi|)\, d\xi \right\} - \pi P^*(\lambda) = 0 \qquad (x > 0) \tag{1.9}$$

The integral Equation (1.9) will be solved by the method of Fok [2]. We shall dwell here on the main outline of the solution. Applying to (1.9) the Laplace transform with respect to $x$, we obtain

$$\frac{\Phi(k, \lambda)}{\sqrt{\lambda + k}} + G(k, \lambda) = \frac{\sqrt{\lambda - k}}{\pi} \int_\lambda^\infty \frac{\Phi(\xi, \lambda)}{(\xi - k)\sqrt{\xi^2 - k^2}}\, d\xi \qquad (0 < \operatorname{Re} k < \operatorname{Re}\lambda) \tag{1.10}$$

Here

$$\Phi(k, \lambda) = \int_0^\infty e^{-kx} \varphi(x, \lambda)\, dx$$

$$G(k, \lambda) = \sqrt{\lambda - k} \left[ \frac{1}{\pi} \int_\lambda^\infty \frac{\Phi(\xi, \lambda) e^{-2\xi}}{(\xi + k)\sqrt{\xi^2 - \lambda^2}}\, d\xi - \frac{P^*(\lambda)}{k\lambda} + \frac{V^*(\lambda)}{\pi} \int_\lambda^\infty \frac{1 - e^{-2\xi}}{\xi(\xi + k)\sqrt{\xi^2 - k^2}}\, d\xi \right]$$

In order that the solution of Equation (1.7) be bounded for $|x| > 1$ and $t > 0$ and integrable with regard to $x$ in an arbitrary final interval, it is necessary that the function $\Phi(k, \lambda)$ be regular for $\operatorname{Re} k > 0$ and that it tend toward zero for $k \to \infty$. Then, the function $G(k, \lambda)$ is regular in the strip $0 < \operatorname{Re} k < \operatorname{Re} \lambda$ and tends toward zero for $k \to \infty$. Thus, this function can be expressed by means of a Cauchy integral, the contour of which can be so deformed that it almost fully embraces the strip of regularity

$$G(k, \lambda) = -\frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \frac{G(\xi, \lambda)}{\xi - k}\, d\xi + \frac{1}{2\pi i} \int_{\lambda - i\infty}^{\lambda + i\infty} \frac{G(\xi, \lambda)}{\xi - k}\, d\xi = G_1(k, \lambda) + G_2(k, \lambda) \tag{1.11}$$

where

$$G_1(k,\lambda) = -\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{G(\xi,\lambda)}{\xi-k}\,d\xi = \frac{1}{\pi}\int_{\lambda}^{\infty}\frac{\Phi(\xi,k)\,e^{-2\xi}}{(\xi+k)\sqrt{\xi-\lambda}}\,d\xi - \frac{P^*(\lambda)}{k\sqrt{\lambda}} +$$
$$+\frac{V^*(\lambda)}{\pi}\int_{\lambda}^{\infty}\frac{1-e^{-2\xi}}{\xi(\xi+k)\sqrt{\xi-\lambda}}\,d\xi \qquad (1.12)$$

is a regular function in a semi-plane Re $k > 0$ and function $G_2(k, \lambda)$ is regular in a semi-plane Re $k <$ Re $\lambda$.

Substituting (1.11) into (1.10) we obtain

$$\frac{\Phi(k,\lambda)}{\sqrt{\lambda+k}} + G_1(k,\lambda) = \frac{\sqrt{\lambda-k}}{\pi}\int_{\lambda}^{\infty}\frac{\Phi(\xi,\lambda)}{(\xi-k)\sqrt{\xi^2-\lambda^2}}\,d\xi - G_2(k,\lambda) \qquad (1.13)$$

In (1.13) the left part is regular in a semi-plane Re $k > 0$ and the right part is regular in a semi-plane Re $k <$ Re $\lambda$ and both parts tend toward zero for $k \to \infty$. Consequently, both parts of Equation (1.13) equal zero. Thus, considering (1.12) we obtain

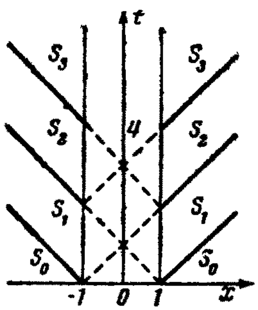


Fig. 2.

$$\Phi(k,\lambda) = -\sqrt{\lambda+k}\,G_1(k,\lambda) =$$
$$= \frac{P^*(\lambda)}{k\sqrt{\lambda}} - \frac{V^*(\lambda)}{\pi}\int_{\lambda}^{\infty}\frac{1-e^{-2\xi}}{\xi(\xi+k)\sqrt{\xi-\lambda}}\,d\xi -$$
$$-\frac{1}{\pi}\int_{\lambda}^{\infty}\frac{\Phi(\xi,\lambda)\,e^{-2\xi}}{(\xi+k)\sqrt{\xi-\lambda}}\,d\xi \qquad (\mathrm{Re}\,k > 0) \qquad (1.14)$$

Using the inverse Laplace transform, first with respect to $k$ and then with respect to $\lambda$ we obtain from (1.14) the velocity distribution for $z = 0$ in the variables $x$, $t$

$$v(x,t) = V(t) \ \text{ in } S, \qquad v(x,t) = P(t) \ \text{ in } S_0$$
$$v(x,t) = P(t) + \frac{1}{\pi\sqrt{|x|-1}}\int_{|x|-1}^{t}[P(t-\tau) - V(t-\tau)]\,\frac{\sqrt{\tau-|x|+1}}{\tau}\,d\tau \ \text{ in } S_1 \qquad (1.15)$$

and for the velocity in the remaining domains we obtain a recurrence formula

$$v(x,t)=P(t)+\frac{1}{\pi\sqrt{|x|-1}}\left(\int\limits_{|x|-1}^{t}P(t-\tau)\frac{\sqrt{\tau-|x|+1}}{\tau}d\tau-\right. \qquad (1.16)$$

$$\left.-\int\limits_{|x|-1}^{|x|+1}V(t-\tau)\frac{\sqrt{\tau-|x|+1}}{\tau}d\tau-\int\limits_{|x|+1}^{t}v(\tau-|x|,t-\tau)\frac{\sqrt{\tau-|x|+1}}{\tau}d\tau\right) \text{ in } S_n$$

where the function $v(x, t)$ under the integral is known if the solution has been found in domains $S_1$, $S_2$, ..., $S_{n-1}$ (Fig. 2).

For the case of a fixed plate one should put in (1.16) $V(t) \equiv 0$ and in this case one may consider the problem solved. The pressure within liquid is then calculated from (1.4) and the pressure drop across the plate is determined from (1.5).

In the case of a moving plate one has to set up the equation of the plate motion and having solved it one must determine the function $V(t)$. This will be the subject of the present discussion.

**2.** We shall now calculate the distribution of the pressure drop on the plate for $x > 0$. For the initial period of time, depending upon the values of $x$ and $q$ three different cases are possible (Fig. 3).

The first case $0 \leqslant x \leqslant 1 - t$, $t \leqslant 1$ (Fig. 3a)

$$\Delta p_1=2\left[P(t)-\frac{1}{\pi}\frac{\partial}{\partial t}\iint\limits_{\sigma}\frac{V(\tau)\,d\xi d\tau}{\sqrt{(t-\tau)^2-(x-\xi)^2}}\right]=2\,[P(t)-V(t)] \qquad (2.1)$$

The second case $1 - t \leqslant x \leqslant 1$, $t \leqslant 1$ and $t - 1 \leqslant x \leqslant 1$, $t > 1$ (Fig.3b)

$$\Delta p_2=2\left\{P(t)-\frac{1}{\pi}\frac{\partial}{\partial t}\left[\iint\limits_{\sigma+\sigma'}\frac{V(\tau)\,d\xi\,d\tau}{\sqrt{(t-\tau)^2-(x-\xi)^2}}+\iint\limits_{\sigma_0+\sigma_1}\frac{P(\tau)\,d\xi\,d\tau}{\sqrt{(t-\tau)^2-(x-\xi)^2}}+\right.\right.$$

$$\left.\left.+\iint\limits_{\sigma_1}\frac{f(\xi,\tau)\,d\xi\,d\tau}{\sqrt{(t-\tau)^2-(x-\xi)^2}}\right]\right\}$$

Here

$$f(x,t)=\frac{1}{\pi\sqrt{x-1}}\int\limits_{x-1}^{t}[P(t-\tau)-V(t-\tau)]\frac{\sqrt{\tau-x+1}}{\tau}d\tau$$

Introducing characteristic coordinates

$$x_1=t+x, \qquad t_1=t-x$$

one can show that

$$\iint\limits_{\sigma_1}\frac{f(\xi,\tau)\,d\xi\,d\tau}{\sqrt{(t-\tau)^2-(x-\xi)^2}}=\iint\limits_{\sigma'}\frac{[P(\tau)-V(\tau)]\,d\xi d\tau}{\sqrt{(t-\tau)^2-(x-\xi)^2}} \qquad (2.2)$$

Taking into account (2.2) one obtains after formidable calculations

$$\Delta p_2 = 2\left(P(t) - V(t) - \frac{1}{\pi}\frac{\partial}{\partial t}\int_0^{t-(1-x)} [P(\tau) - V(\tau)]\left[\frac{\pi}{2} + \arcsin\left(1 - 2\,\frac{1-x}{t-\tau}\right)\right]d\tau\right) \tag{2.3}$$

The third case $0 \leqslant x \leqslant t - 1$, $t \geqslant 1$ (Fig. 3c). In analogy with the previous case we obtain

$$\Delta p_3 = 2\left\{P(t) - V(t) - \frac{1}{\pi}\frac{\partial}{\partial t}\int_0^{t-(1-x)} [P(\tau) - V(\tau)]\left[\frac{\pi}{2} + \arcsin\left(1 - 2\,\frac{1-x}{t-\tau}\right)\right]d\tau - \right.$$

$$\left. - \frac{1}{\pi}\frac{\partial}{\partial t}\int_0^{t-(1+x)} [P(\tau) - V(\tau)]\left[\frac{\pi}{2} + \arcsin\left(1 - 2\,\frac{1+x}{t-\tau}\right)\right]d\tau\right\} \tag{2.4}$$

We shall note that in (2.1), (2.3) and (2.4) the term $2P(t)$ expresses the pressure on a stationary infinite plate, the term $2V(t)$ is the change of the pressure drop resulting from the displacement of the plate and the integral expressions account for the diffraction effects on the edges.

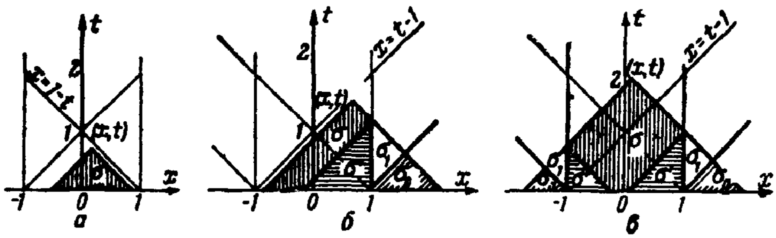


Fig. 3.

The force acting on a unit length of the plate is

$$F(t) = 2\int_0^1 \Delta p\,dx = 2\left[\int_0^{1-t} \Delta p_1 dx + \int_{1-t}^1 \Delta p_2\,dx\right] \qquad \text{for } 0 \leqslant t \leqslant 1$$

$$F(t) = 2\int_0^1 \Delta p\,dx = 2\left[\int_0^{t-1} \Delta p_3\,dx + \int_{t-1}^1 \Delta p_2 dx\right] \qquad \text{for } 1 \leqslant t \leqslant 2$$

Skipping the calculations we obtain in both cases

$$F(t) = 4\left(P(t) - V(t) - \frac{1}{2}\int_0^t [P(\tau) - V(\tau)]\,d\tau\right) \qquad (0 \leqslant t \leqslant 2) \tag{2.5}$$

For the time $t > 2$ we obtain accordingly

$$F(t) = 4\left(P(t) - V(t) - \frac{1}{2}\int_{2n}^t [P(t) - V(\tau)]\,d\tau - R_n(t)\right) \qquad \begin{pmatrix} 2n \leqslant t \leqslant 2n+2 \\ n = 1, 2, \ldots \end{pmatrix} \tag{2.6}$$

Here

$$R_n(t) = \frac{1}{\pi}\frac{\partial}{\partial t}\left[\int_0^1 dx \int_0^{2n} d\tau \int_{\xi_1}^{\xi_2} \frac{v(\xi,\tau)\,d\xi}{\sqrt{(t-\tau)^2-(x-\xi)^2}} + \int_0^{x_2} dx \int_{2n}^{\tau_1} d\tau \int_{\xi_3}^{\xi_4} \frac{v(\xi,t)-P(\tau)}{\sqrt{(t-\tau)^2(x-1+\xi)^2}}\,d\xi\right]$$

$$\xi_1 = x-(t-\tau), \quad \tau_1 = (t+2n-x)/2, \quad \xi_3 = \tau+1-2n$$
$$\xi_2 = x+(t-\tau), \quad x_2 = t-2n, \quad \xi_4 = 1-x+t-\tau$$

is a known function if one has found the solution for $t \leqslant 2n$.

Assume that the plate is subjected only to hydrodynamic pressure. Newton's law in dimensionless variables will have the form

$$V'(t) = \frac{\varepsilon}{4}F(t) \qquad \left(\varepsilon = \frac{l\rho_0}{h\rho}\right) \tag{2.7}$$

where $h$ is the thickness and $\rho$ is the density of the plate.

Using (2.5) and (2.6) and differentiating (2.7) once with regard to $t$, we obtain

$$V''(t) + \varepsilon V'(t) - \frac{1}{2}\varepsilon V(t) = \varepsilon\left[P'(t) - \frac{1}{2}P(t)\right] \qquad (0 \leqslant t \leqslant 2) \tag{2.8}$$
$$V''(t) + \varepsilon V'(t) - \frac{1}{2}\varepsilon V(t) = \varepsilon\left[P'(t) - \frac{1}{2}P(t) - R_n'(t)\right] \qquad \begin{pmatrix}2n \leqslant t \leqslant 2n+2\\ n=1,2,\ldots\end{pmatrix}$$

with the conditions

$$V(0) = 0, \qquad V'(0) = \varepsilon P(0)$$
$$V = V(2n), \qquad V' = \varepsilon\,[P(2n) - V(2n) - R_n(2n)] \qquad \text{for } t = 2n \tag{2.9}$$

where $V(2n)$ and $R_n(2n)$ are known if the solution for $t \leqslant 2n$ has been found.

Solving these equations we obtain (2.10)

$$V(t) = \frac{\varepsilon}{\lambda_1-\lambda_2}\int_0^t P(t-\tau)\left[\left(\lambda_1-\frac{1}{2}\right)e^{\lambda_1\tau} - \left(\lambda_2-\frac{1}{2}\right)e^{\lambda_2\tau}\right]d\tau \qquad (0 \leqslant t \leqslant 2)$$

$$V(t) = \frac{\varepsilon}{\lambda_1-\lambda_2}\int_0^{t-2n} P(t-\tau)\left[\left(\lambda_1-\frac{1}{2}\right)e^{\lambda_1\tau}\left(\lambda_2-\frac{1}{2}\right)e^{\lambda_2\tau}\right]d\tau + \frac{V(2n)}{\lambda_1-\lambda_2}[\lambda_1 e^{\lambda_1(t-2n)} -$$
$$- \lambda_2 e^{\lambda_2(t-2n)}] - \frac{\varepsilon}{\lambda_1-\lambda_2}\int_0^{t-2n} R_n(t-\tau)(\lambda_1 e^{\lambda_1\tau} - \lambda_2 e^{\lambda_2\tau})\,d\tau \qquad \begin{pmatrix}2n \leqslant t \leqslant 2n+2\\ n=1,2,\ldots\end{pmatrix}$$

where

$$\lambda_{1,2} = -(\varepsilon \mp \sqrt{\varepsilon^2+2\varepsilon})/2$$

Thus, the velocity of the plate is determined successively through the time interval $\Delta t = 2$ in accordance with the Formulas (2.10).

Solving equations (2.8) for $P(t)$ we obtain

$$P(t) = \frac{1}{\varepsilon}\left[V'(t) + \left(\varepsilon + \frac{1}{2}\right)V(t) + \frac{1}{4}\int_0^t e^{(t-\tau)/2}V(\tau)\,d\tau\right] \qquad (0 \leqslant t \leqslant 2)$$

$$P(t) = \frac{1}{\varepsilon}\left\{V'(t) - V'(2n) + \left(\varepsilon + \frac{1}{2}\right)[V(t) - V(2n)] + \varepsilon[R_n(t) - R_n(2n) + \right.$$

$$\left. + \frac{1}{4}\int_{2n}^t e^{(t-\tau)/2}[V(\tau) - 2\varepsilon R_n(\tau)]\,d\tau\right\} \qquad \binom{2n \leqslant t \leqslant 2n+2}{n = 1, 2, \ldots} \tag{2.11}$$

The expressions (2.11) can be utilized to determine in acoustic approximation the profile of the impinging wave if experimental data about the motion of the plate are known.

We consider now the plate in the initial time period $0 < t < l/c$.

Returning to dimensional variables in (2.10) we obtain

$$V(t) = \frac{1}{\rho h}\int_0^t P(t-\tau)\left[\frac{\sqrt{\varepsilon^2 + 2\varepsilon} - \varepsilon - 1}{\sqrt{\varepsilon^2 + 2\varepsilon}}\exp\left(\frac{2\lambda_1 c}{l}\tau\right) + \right.$$

$$\left. + \frac{\sqrt{\varepsilon^2 + 2\varepsilon} + \varepsilon + 1}{\sqrt{\varepsilon^2 + 2\varepsilon}}\exp\left(\frac{2\lambda_2 c}{l}\tau\right)\right]d\tau \tag{2.12}$$

Let us assume that the wave is short. Then its action can be treated as an impulsive impact. Denoting by $I$ the specific impulse of the wave we obtain from (2.12)

$$V(t) = \frac{I}{\rho h}\left[\frac{\sqrt{\varepsilon^2 + 2\varepsilon} - \varepsilon - 1}{\sqrt{\varepsilon^2 + 2\varepsilon}}\exp\left(\frac{2\lambda_1}{l}t\right) + \frac{\sqrt{\varepsilon^2 + 2\varepsilon} + \varepsilon + 1}{\sqrt{\varepsilon^2 + 2\varepsilon}}\exp\left(\frac{2\lambda_2}{l}t\right)\right]$$

We observe that $V(t) = 0$ at a time

$$t^* = \frac{l}{c}\frac{\ln(\sqrt{\varepsilon^2 + 2\varepsilon} + \varepsilon + 1)}{\sqrt{\varepsilon^2 + 2\varepsilon}}, \qquad t^* \leqslant \frac{l}{c}, \qquad \frac{dt^*}{d\varepsilon} < 0 \tag{2.13}$$

It follows that in the case of a short lasting wave, there is a moment during the passage of the diffraction wave from one edge of the plate to the other, when the velocity of the plate decreases to zero and then it changes its sign. Obviously, at that time the plate will suffer the largest displacement

$$u_{\max} = \int_0^{t^*} V(\tau)\,d\tau = \frac{Il}{\rho hc\varepsilon}\left[1 - (\sqrt{\varepsilon^2 + 2\varepsilon} + \varepsilon + 1)^{-\varepsilon/\sqrt{\varepsilon^2+2\varepsilon}}\right]$$

Let us assume that the wave has a steady profile $P = P_0$. Then it follows from (2.12)

$$V_{\max} = V(t^*) = \frac{P_0 l}{\rho hc\varepsilon}[1 - (\sqrt{\varepsilon^2 + 2\varepsilon} + \varepsilon + 1)^{-\varepsilon/\sqrt{\varepsilon^2+2\varepsilon}}]$$

where $t^*$ is the same as in (2.13). It follows that in this case the acceleration changes its sign at time $t^*$.

From the above cases one can infer that also in the case of any other arbitrary pressure profiles satisfying the condition $P'(t) \leqslant 0$ the plate velocity and acceleration change their sign during the time span $t \sim l/c$ This result can be related to the strong resistance of the liquid and of the diffraction to the motion of the plate. One can expect that subsequently the plate will gradually slow down its motion.

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